1. Inno, GATIN, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, inno.gatin@fsb.hr (corresponding author)
2. Vuko, VUKČEVIĆ, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, vuko.vukcevic@fsb.hr
3. Hrvoje, JASAK, Faculty of Mechanical Engineering and Naval Architecture, University of Zagreb, Ivana Lučića 5, hrvoje.jasak@fsb.hr

COUPLING OF HIGHER ORDER SPECTRAL METHOD AND COMPUTATIONAL FLUID DYNAMICS

Abstract

This paper presents an efficient method for simulating extreme wave loads using Higher Order Spectral (HOS) method and Computational Fluid Dynamics (CFD) in OpenFOAM. HOS is capable of nonlinear propagation of arbitrary wave spectrum, while being computationally effective. CFD enables fully nonlinear two-phase, turbulent flow solution with vorticity effects at the cost of increased CPU time. In this work the coupling of HOS and CFD is briefly presented and numerical simulations depicting the capabilities of the coupling are shown.

Key words in English: Extreme Wave Loads Simulation, Computational Fluid Dynamics, Higher Order Spectral Method, OpenFOAM

SPREGA SPEKTRALNE METODE VIŠIH REDOVA I RAČUNALNE DINAMIKE FLUIDA

Sažetak

U ovom radu je pokazana mogućnost efikasnog provođenja numeričkih simulacija opterećenja konstrukcija uslijed nailaska ekstremnog vala u programu OpenFOAM. Spregom spektralne potencijalne nelinearne metode viših redova za propagaciju površinskih valova i računalne dinamike fluida omogućuje se provođenje ovakvih simulacija u razumnom vremenu. Spektralna metoda viših redova (eng. "*Higher Order Spectral Method",* HOS) omogućuje nelinearnu propagaciju proizvoljnog valnog spektra uz vrlo niske zahtjeve proračunskog vremena, dok računalna dinamika fluida pruža potpuno rješavanje dvofaznog, turbulentnog i vrtložnog strujanja fluida. Ukratko je prikazan način sprege HOS metode i računalne dinamike fluida, te primjer simulacije opterećenja konstrukcije uslijed nailaska ekstremnog vala.

Ključne riječi na hrvatskom: ekstremna valna opterećenja, računalna dinamika fluida, spektralna metoda viših redova, OpenFOAM

1. Introduction

There is an increasing tendency in the field of naval and offshore engineering towards the use of RANS (Raynolds Averaged Navier-Stokes) based CFD methods for simulating wave related phenomena. The main reason for simulating waves is to assess wave loads exerted on marine structures. Specifically, there is a raising concern for the safety of naval and offshore objects encountered by extreme waves. To obtain a realistic extreme wave it is necessary to propagate an irregular sea state for a sufficient amount of time on a large domain in order to capture its random emergence. The main disadvantage of RANS based methods is the high computational demand, which currently prevents their application for long-time evolution of realistic wave fields.

In this paper a framework for efficient extreme wave simulations by coupling HOS ([1], [2], [3], [4]) potential flow method and RANS based CFD is presented. Combining the low cost HOS method and fully nonlinear, two-phase, turbulent CFD, their advantages can be exploited while avoiding their individual disadvantages. The framework relies on HOS to perform a nonlinear long-time irregular sea state propagation on a large domain to obtain a statistically and physically realistic freak wave. Due to the spectral nature of the HOS method, the free surface elevation and the velocity field can be fully reconstructed and used in a smaller CFD domain. With the initial wave field provided by HOS, CFD can fully nonlinearly simulate the extreme wave in the time domain, enabling fluid-structure interaction simulations. The coupling is performed in open source computational continuum mechanics (CCM) software OpenFOAM [5].

This paper is organised as follows. In section 2, the mathematical model of HOS is outlined, and the coupling with CFD is explained. The third section shows the details of the numerical method, focusing on the CFD method and the HOS-CFD coupling. In section 4, a brief validation of HOS is shown to confirm the accuracy of the implemented algorithm. In section 5, two 3D extreme wave simulations are shown. Finally, a short conclusion is given.

2. Mathematical model

In this section a brief overview of the HOS mathematical model and the coupling of HOS and CDF is given.

2.1. Higher Order Spectral method

HOS is a pseudo-spectral method for solving nonlinear partial differential equations describing the dynamic and kinematic free surface boundary conditions in potential flow:

- Dynamic free surface boundary condition:

$$\frac{\partial \phi}{\partial t} + gz + \frac{1}{2}(\nabla \phi)^2 = 0, \tag{1}$$

  where ϕ is the velocity potential, g is the gravitational acceleration, z is the vertical coordinate (negative below the calm free surface), and t is the time variable.

- Kinematic free surface boundary condition:

$$\frac{\partial \eta}{\partial t} + \nabla_h \phi \cdot \nabla_h \eta = \frac{\partial \phi}{\partial z}, \tag{2}$$

  where η stands for free surface elevation, and index h denotes horizontal gradient.

Equations (1) and (2) represent nonlinear boundary conditions for the problem defined by the Laplace equation, which stems from the potential flow mass conservation law. In HOS method, a Fourier series expansion is assumed as a shape function for the velocity potential ϕ which satisfies the Laplace equation:

$$\phi(x,y,z,t) = \sum_k \sum_l c_{k,l}(t) \frac{\cosh\left(K_{k,l}(z+d)\right)}{\cosh\left(K_{k,l}d\right)} e^{iK_k x} e^{iK_l y}, \tag{3}$$

where $c_{k,l}(t)$ presents the unknown time-dependent Fourier coefficients, d is a constant depth, $K_{k,l}$ is the absolute value of the wave number, K_k is the x direction wave number, while K_l is the y direction wave number. The spatial derivatives in equations (1) and (2) are assessed in spectral domain, while the temporal integration is performed in physical time. The data is projected back and forth between the spatial and spectral domain using Fast Fourier Transform (FFT) algorithm, rendering the method efficient. The nonlinearities in equations (1) and (2) are resolved sequentially by expanding the velocity potential in a perturbation series with respect to wave steepness ε, which is a measure of nonlinearity of the wave field:

$$\phi(x,y,z,t) = \phi_1 + \varepsilon\phi_2 + \varepsilon^2\phi_3 + \cdots = \sum_{m=1}^{M} \phi^{(m)}, \tag{4}$$

where M is the order of nonlinearity of the perturbation series, and $\phi^{(m)} = \varepsilon^{m-1}\phi_m$. The individual orders of velocity potential $\phi^{(m)}$ are expanded in Taylor series around the undisturbed free surface ($z = 0$):

$$\phi(x,y,\eta(x,y,t)\,,t) = \psi(x,y,t) = \sum_{m=1}^{M} \sum_{i=1}^{M-m} \frac{\eta^i}{i!} \frac{\partial^i}{\partial z^i} \phi^{(m)}(x,y,0,t), \tag{5}$$

where ψ stands for the surface velocity potential. The individual velocity potentials are then determined by equating equation (5) order wise. Once the orders of velocity potential are known, their vertical derivatives needed in equations (1) and (2) can be calculated sequentially in the same manner, see [1].

Following West et al. [1], the order consistency is retained with respect to ε while performing the multiplications in equations (1) and (2) by truncating the vertical derivative of ϕ at the appropriate order of nonlinearity m in equation (4).

The initial condition has to be defined for ψ and η. Linear solution of equations (1) and (2) is the most convenient choice, since a variety of regular and irregular wave fields can easily be modelled. However, using a linear solution to initialize a nonlinear wave field causes unstable evolution of the system. To stabilise the simulation, a time relaxation initialisation scheme proposed by Dommermuth [6] is employed.

2.2. HOS-CFD coupling

One way coupling is performed where the free surface elevation and velocity field are defined in CFD using the solution obtained with HOS method. The velocity field is determined by deriving the velocity potential in x, y and z direction:

$$v_x(x,y,z,t) = \sum_k \sum_l c_{k,l}(t) iK_k \frac{\cosh\left(K_{k,l}(z'+d)\right)}{\cosh\left(K_{k,l}d\right)} e^{iK_k x} e^{iK_l y}, \tag{6}$$

$$v_y(x,y,z,t) = \sum_k \sum_l c_{k,l}(t) iK_l \frac{\cosh\left(K_{k,l}(z'+d)\right)}{\cosh\left(K_{k,l}d\right)} e^{iK_k x} e^{iK_l y}, \tag{7}$$

$$v_z(x,y,z,t) = \sum_k \sum_l c_{k,l}(t) K_{k,l} \frac{\sinh\left(K_{k,l}(z'+d)\right)}{\cosh\left(K_{k,l}d\right)} e^{iK_k x} e^{iK_l y}, \tag{8}$$

where z' stands for the altered z coordinate using the Wheeler correction:

$$z' = qz + d(q-1), \tag{9}$$

here $q = d/(d+\eta)$. The Wheeler correction is needed since ϕ is determined for an approximate position of the free surface at $z = 0$ instead of $z = \eta$. The free surface is evaluated directly from the Fourier series representing the free surface elevation obtained using FFT.

3. Numerical method

In order to facilitate HOS-CFD coupling, HOS method is implemented in the Naval Hydro pack based on OpenFOAM. OpenFOAM uses an arbitrary polyhedral finite volume method with unstructured grid support [5]. An incompressible, two phase, turbulent flow model is used. The coupling of potential flow and CFD is achieved using SWENSE (Spectral Wave Explicit Navier-Stokes Equation) method, which uses a solution decomposition approach where only the difference between the incident potential flow and the full CFD solution is solved for. Implicitly redistanced Level Set method is used for interface capturing [7]. The discontinuity of the pressure gradient and density at the interface is modelled exactly using the embedded free surface approach, which eliminates spurious air velocities often encountered in CFD models using conditionally averaged equations. The computational domain is decomposed in two regions: full CFD region, and the blending region near the far field boundaries where the HOS solution is used. This enables a reduced CFD domain where the wave field is prescribed by HOS near the boundaries, while fully nonlinear flow solution is obtained in the vicinity of the structure. The present CFD numerical model is validated for seakeeping and steady resistance, the reader is referred to [8], [9] and [10] for details.

HOS simulation may be performed either alongside CFD simulation to provide a potential flow solution in each time step, or separately as an individual simulation. Equations (1) and (2) are integrated in time using fifth-order Cash-Karp embedded Runge-Kutta scheme with error control and adjustable time-step size.

4. HOS validation

Following Dommermuth [6], a comparison of higher order wave components (up to 8th order) of a propagating monochromatic wave is conducted between the nonlinear analytical Stokes solution and a long-time HOS simulation. HOS simulation is initialized using a linear solution. **Table 1** shows harmonic modal amplitudes of the evolved higher order wave components and the relative error with respect to the Stokes analytical solution. As it can be seen the relative errors are very small, only 4.34×10^{-6} % for the first order. The largest error is only -1.09×10^{-1} % for the 8th order, which is of negligible importance due to its small amplitude.

Table 1. Higher order wave components comparison.

Tablica 1. Usporedba amplituda valnih harmonika.

Order	Harmonic component amplitude, m		Relative error, %
	Analytical solution	HOS solution	
1	9.9870520×10^{-2}	9.9870524×10^{-2}	4.34×10^{-6}
2	5.0594125×10^{-3}	5.0594197×10^{-3}	1.43×10^{-4}
3	3.8584235×10^{-4}	3.8584235×10^{-4}	2.78×10^{-4}
4	3.4929691×10^{-5}	3.4929838×10^{-5}	4.20×10^{-4}
5	3.4769679×10^{-6}	3.4769678×10^{-6}	-3.26×10^{-6}

6	3.6763951×10^{-7}	3.6763189×10^{-7}	2.07×10^{-3}
7	4.0531740×10^{-8}	4.0530830×10^{-8}	-2.24×10^{-3}
8	4.6076934×10^{-9}	4.6026818×10^{-9}	-1.09×10^{-1}

5. Simulations

Two full scale simulations are performed to demonstrate the capability of the proposed coupling. JONSWAP directional wave energy spectrum with the significant wave height $H_s = 10.5$ m and peak period $\mathrm{T}_P = 9.5$ s is used to define a three dimensional initial wave field. The wave field is propagated using HOS for 1500 s on 1100×1100 m domain. **Fig. 1** shows the free surface elevation in the HOS simulation. Extreme wave, shown on **Fig. 2**, emerged at $t = 532$ s with the wave height $H = 21.9$ m, and was used in both simulations. The first simulation shows an extreme wave encountering a barge, while the second simulation shows an oblique extreme wave impact upon a KRISO Container Ship (KCS). Both simulations were performed in full scale.

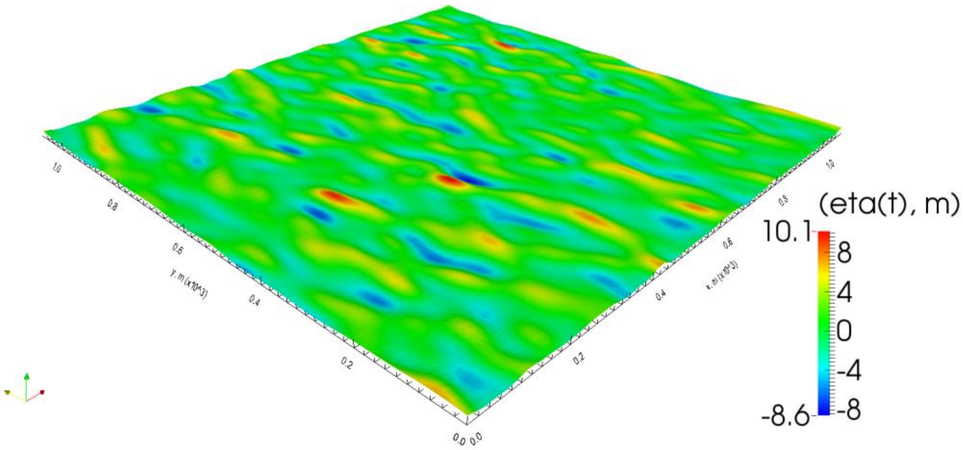

Fig. 1. 3D wave field in the HOS simulation.

Slika 1. Trodimenzionalno polje valova u HOS simulaciji.

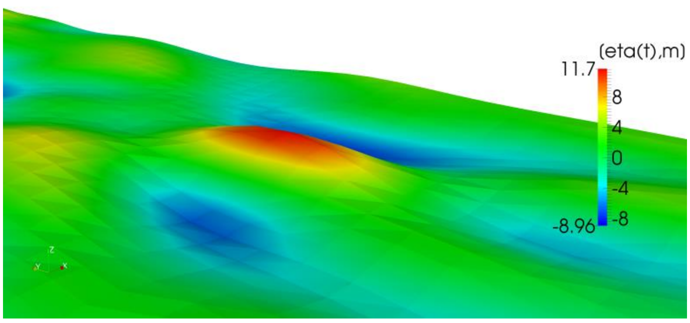

Fig. 2. Close up view of the naturally emerging freak wave in the HOS simulation at t = 532 s.

Slika 2. Uvečani prikaz prirodno nastalog ekstremnog vala u HOS simulaciji u trenutku t = 532 s.

5.1. Extreme wave impact on a barge

The freely floating barge has been simulated with all six degrees of freedom. The dimensions of the barge are $L \times B \times D = 27 \times 7 \times 5$ m, with draft $T = 2.5$ m. Domain size is $123 \times 86 \times 23$ m, discretised with $110\,000$ cells. The simulation took 37 hours of CPU time for 25 seconds of simulated time on a four core 2.4 GHz processor. **Fig. 3** shows the encounter of the extreme wave on the barge sequentially, where significant green sea effects can be seen.

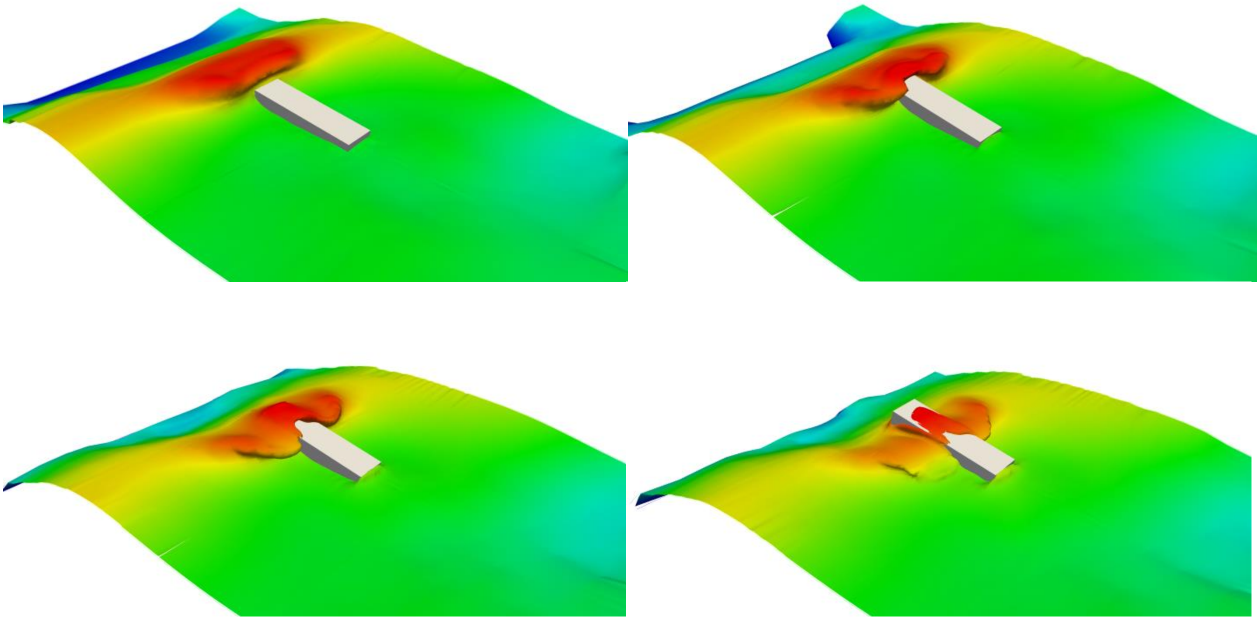

Fig. 3 Extreme wave encountering the barge; t = 11, 11.5, 12 and12.5 s.

Slika 3. Nailazak ekstremnog vala na baržu; t = 11, 11.5, 12 i 12.5 s.

5.2. Extreme wave impact on a full-scale KCS

A full scale KCS was simulated without forward speed with six degrees of freedom. Dimensions of the ship are $L \times B \times D = 230 \times 32.2 \times 19$ m with the draft of $T = 10.8$ m. Grid with 1.2 million cells is used to discretise the domain of size $860 \times 600 \times 530$ m. In this case the mean direction of spectrum propagation is oblique with respect to the ship heading by 25°. The simulation required 132 hours of CPU time to simulate 30 s of simulated time on a four core 3.7 GHz processor. **Fig. 4** shows the encounter of the extreme wave on the KCS sequentially.

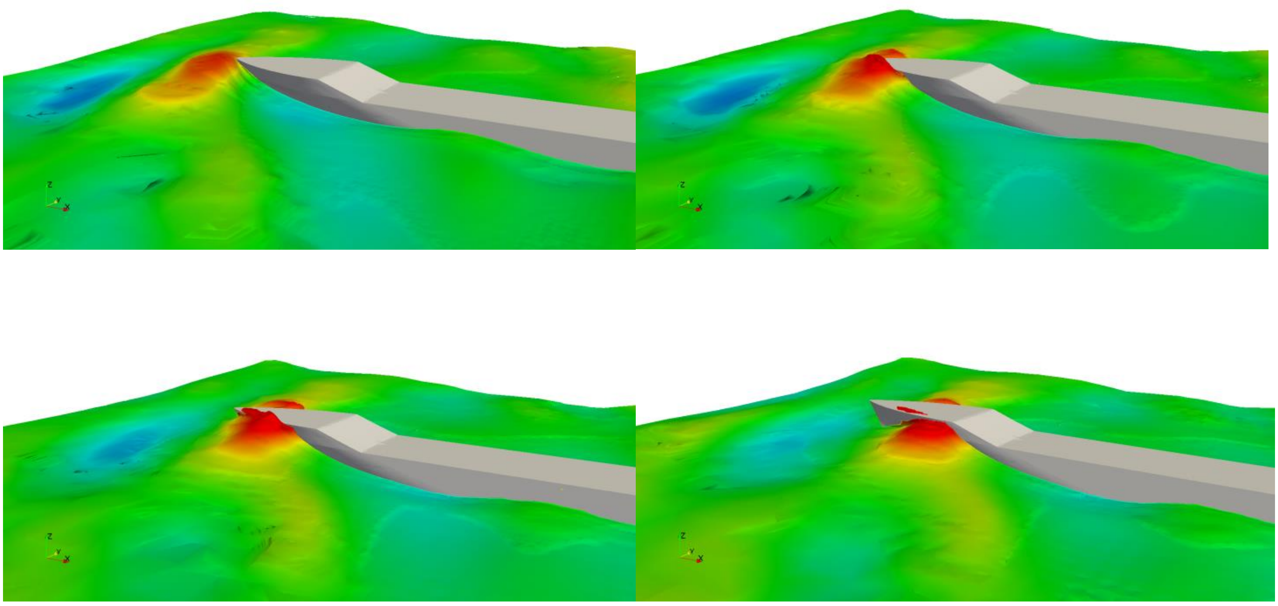

Fig. 4 Extreme wave encountering the KCS at t = 5, 6, 7 and 8 s.

Slika 4. Nailazak ekstremnog vala na KCS u vremenskim trenutcima t = 5, 6, 7 i 8 s.

6. Conclusion

In this paper an efficient method for simulating extreme wave loads by HOS-CFD coupling presented.

A short validation of the HOS algorithm is presented to confirm the accuracy of the implemented algorithm. The validation is performed by evaluating the higher order nonlinear wave components emerging during monochromatic wave propagation. The validation shows satisfying accuracy of the implemented algorithm.

Finally, two simulations are shown do depict the capability of the coupling. The first is a simulation of an extreme wave impact on a full scale freely floating barge. The second simulation shows an oblique extreme wave impact on a full scale container ship with six degrees of freedom.

HOS algorithm and CFD model have been successfully validated separately, which leads to a conclusion that the proposed coupling is also accurate. Nonetheless, the coupling has to be validated to prove its accuracy for the calculation of extreme wave loads. Unfortunately, no experimental data is available at present time.

List of references:

[1] WEST, B.J., BRUECKNER, K.A., JANDA, A.: "A New Numerical Method for Surface Hydrodynamics", Journal of Geophysical Research. 92(1987), p. 11803-11824.

[2] DOMMERMUTH, D.G., YUE, D.K.P.: "A high-order spectral method for the study of nonlinear gravity waves", J. Fluid Mech. 184(1987), p. 267-288.

[3] DUCROZET, G., BONNEFOY, F., LE TOUZE, D., FERRANT, P.: "3-D HOS simulations of extreme waves in open seas", Nat. Hazards Earth Syst. Sci. 7(2007), p. 109-122.

[4] TANAKA, M.: "A method of studying nonlinear random gravity waves by direct numerical simulation", Fluid Dyn. Res. 28(2001), p. 41-60.

[5] JASAK, H.: "OpenFOAM: Open Source CFD in Research and Industry", International Journal of Naval Architecture and Ocean Engineering. 2(2009), p. 89-94.

[6] DOMMERMUTH, D.G.: "The initialization of nonlinear waves using an adjustment scheme", Wave Motion. 32(2000), p. 307-317.

[7] SUN, Y., BECKERMANN, C.: "Sharp interface tracking using the phase–field equation", J. Comput. Phys. 220(2007), p. 626-653.

[8] GATIN I, JASAK, H, VUKČEVIĆ, V.: "Validation and Verification of Steady Resistace KCS Simulations with Sinkage and Trim using Embedded Free Surface Method", In Tokyo 2015: A workshop on CFD in Ship Hydroyinamics; 2015

[9] VUKČEVIĆ, V., JASAK, H.: "Seakeeping Validation and Verification using Decomposition Model based on Embedded Free Surface Method", In Tokyo 2015: A workshop on CFD in Ship Hydroyinamics; 2015.

[10] VUKČEVIĆ J, JASAK H.: "Validation and Verification of Decomposition Model based on Embedded Free Surface Method for Oblique Wave Seakeeping Simulations", In Tokyo 2015: A workshop on CFD in Ship Hydroyinamics; 2015.